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Modeling Monetary Control in an Interest- Free Economy

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ABSTRACT. The paper discusses the role of monetary policy in an interest-free economy by developing a general equilibrium model. The Central Bank can manipulate three basic instruments for monetary control: the reserve ratio, the profit sharing ratio and the equity holding of banks. Thus it achieves its intermediate target pegging the equilibrium rates of return in the economy. Real money demand can be stimulated by raising the reserve ratio for banks. For contraction of money demand the profit sharing ratio can be raised, even though indirectly. The model does not however integrate uncertainty into the analysis or elaborate on inflation and its effects on the real sector.

I. Introduction

In contemporary economies, interest rate plays a prominent role in the allocation of available funds between borrowers and lenders. Interest rates also represent the core for conventional instruments of monetary policy such as the discount rate and open market operations.

However, a key feature of an Islamic economy is the absence of the payment or receipt of any predetermined (fixed) interest rate which is considered usury. How, then, can the economy function without the familiar institution of interest, and how can the Central Bank effectively control money stock in the absence of the traditional instruments? In place of the fixed rate, an Islamic economy organizes its activities on the basis of the profit-loss sharing (PLS) principle. But here too how are the profit-loss

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sharing ratios determined, and what impact do they have on the level of investment, saving, and the overall efficiency of the economy? Such are only some of the important issues facing Islamic economics that many economists have addressed[1].

Our purpose in this paper is to contribute to this debate in the area of monetary control by developing a general equilibrium model for an interest-free economy which emphasizes the role of monetary policy and the mechanism by which it can effectively control key economic variables. In an interesting paper, Khan and Mirakhor (1987) - thereafter, KM - outlined a banking model in an Islamic (interest-free) economy and found no substantive differences in the way monetary policy impacts the economy. Indeed, according to them, the Central Bank can still achieve similar objectives through controlling the activities of profit-based banks. Our model in this paper differs from that of Khan and Mirakhor's in several respects. First, while KM's is a partial equilibrium model focusing, as it does, only on one sector (banking), ours is a general equilibrium model that embodies banks and other sectors in the economy. Second, the KM analysis is a generalization of an IS-LM model, whereas our analysis is a portfolio-choice variant that uses optimization procedures to derive the solutions. Third, compared to KM's restrictive definition of monetary base that ignores public's holding of cash, we expand our measure of the base to incorporate public's behavior. Fourthly, we employ a different set of policy instruments than that used by KM. In particular, instead of banks' equity and the supply of reserves, we focus on changes in reserve ratios and the profit-sharing ratios as key policy instruments. And, fifthly, in contrast to KM, we incorporate the possible effects of zakat on key macroeconomic variables such as the public's money demand and the rate of return on deposits and on equity.

The rest of the paper is organized as follows. Section II outlines the basic assumptions underlying the model. Section III develops the model. Section IV discusses the model solutions and analyzes policy implications. Section V concludes the paper.

II. Basic Assumptions

In this section, we develop a general equilibrium model that incorporates the main features of an interest-free economy The model draws on earlier work particularly those of Khan and Mirakhor (1987), Tobin and Brainard (1967) and Bernanke and Gertler (1986). It is a portfolio selection model whose main focus is on monetary control in the absence of interest-based transactions.

At the outset, we utilize the following simplifying assumptions:

1. For example, see **Ahmad** (1980), **Ahmed** et al. (1983), **Hasan** (1985), **Khan** and **Mirakhor** (1987), and **Chapra** (1992).

(1) The economy is assumed competitive[2] and consist of three sectors; namely, the households, Islamic (interest-free) banks, and the Central Bank (CB). The economy has three different assets: PLS deposits, banks' equity, and money. Islamic banks are not merely intermediaries, but are participants in real investment activities.

(2) Let there be N identical utility-maximizing households with perfect foresight. At the beginning of the period, the households are endowed with W_t units of nonhuman wealth. At the beginning of each period, households must decide how to allocate their nonhuman wealth between PLS deposits at banks and money. Let M_t denote nominal money holding, and D_t denote the PLS deposits. The households must adhere to the following budget constraint:

$$W_t = D_t + M_t \tag{1}$$

(3) There is a large number of banks that are infinitely-lived and risk-neutral. Each bank has access to (illiquid) investment projects which require specialized evaluation and monitoring technology with a large fixed cost (affordable only to banks). Any information gained from the evaluation of projects remains confidential to the bank. It is assumed that the marginal intermediation cost is proportional to the size of the project. Thus, households can only invest in large projects through the intermediation services of banks since the cost of project evaluation and auditing is prohibitively very large. Each period, the bank can utilize resources at its disposal to identify potentially profitable investment projects. Furthermore, for simplicity, we assume a closed economy model and also that deposits in banks are primarily of the PLS type[3].

(4) The CB holds banks' equity that is publicly observable. This is an incentive compatibility assumption which is needed since banks do not guarantee the nominal values of PLS deposits nor guarantee any fixed (positive) rate of return on them. The proceeds from selling equity are pooled with the PLS funds and invested in *Musharaka* and/or *Mudaraba* projects between the bank, its depositors and the CB. Acquiring large equity funds reduce the risk of insolvency and enhance the willingness of the CB to invest in the banks' projects[4]. The willingness of the CB to invest serves as a signal (information transfer) to PLS deposits' owners regarding the soundness of the bank [See Bashir and Darrat (1992)].

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2. We make this assumption in order to assure that any economic agent (bank or household) is small relative to the market as a whole.

3. Note however that, along with these PLS deposits, other types of deposits (e.g., checking accounts) may also co-exist whereby banks can invest the funds and remunerate their owners out of the generated profits.

4. We assume that banks sell equity to meet capital requirements. Equity shares are traded via open market operations. Only the CB and commercial banks can hold banks' equity. When a bank holds equity of another, it is similar to advancing loans to that bank. However, unlike the (policy-controlled) discount rate, the rate of return on equity is endogenously determined in the market. Note also that when government deposit (Cont'd on next page)

(5) The CB holds banks reserves[5], and also issues currency. Money is the only liability that affects the economy, entering as lump sum transfers.

The objective of the CB is to peg rates of return as intermediate targets and control the monetary base as an operating target. To achieve that, the CB uses bank reserves and the profit-sharing ratio as policy instruments. By manipulating these instruments, the CB can influence the supply of reserves and hence the monetary base. Furthermore, the CB pegs rates of return in the economy by influencing banks' behavior directly through equity holding, and indirectly through reserve changes.

(6) Physical technology is required to convert investment into random output. There is one type of output, and inputs in the production process are capital invested by banks as well as labor (i.e., entrepreneurial skills). Part of the net profit generated from the production process is retained by the bank, and the remainder is paid as dividends to depositors and to the equity holder (i.e., the Central Bank) according to predetermined sharing arrangements.

III. The Model

a. The Bank

Let L_b represent PLS deposits held at a bank, and E_b denote total equity held at the CB. The sum of the PLS deposits and equity are the total liabilities of the bank. The bank keeps some fraction R_b against its PLS deposits with the CB as reserves (required or voluntary) and invests the rest. Let δ be the intermediation cost per unit of capital invested, and K be the level of investment. The purpose of modeling the bank's

(from last page)

insurance exists, it is usually accompanied by regulating minimum (equity) capital requirements for a given amount of intermediation. "Insider" equity is useful in solving the agency problem. In our case, however, we assume that bank's equity is both outside equity (held by CB and other banks) and inside equity (retained earnings). In this paper, we focus on outside equity alone since it evokes close monitoring of banking behavior. By holding equity stake in banks, the CB has a vested interest in monitoring the banks' portfolios. Reducing its equity holding, on the other hand, is not necessarily a sign of bank insolvency. It could merely mean that the respective bank has acquired enough funds to satisfy capital requirements. Nonetheless, to avoid triggering financial crises, the CB should reduce its equity holding only gradually through open market operations. Given its status of "lender of last resort," the CB can prevent insolvencies by advancing PLS loans from the pool of reserves. We thank an anonymous referee for directing our attention to this and many other important issues in our analysis.

5. Many researchers have debated the legality of requiring banks to maintain reserves against PLS deposits. In our model, it is immaterial whether banks keep these reserves at the CB to satisfy legal requirements or do so voluntarily for precautionary purposes. Since the CB can invest these reserves (e.g., in buying equity of banks), it may share the profit thus generated with banks who own the reserves. The reserves on PLS deposits could be postulated as a collateral to overcome the agency problems arising between banks and their depositors, or as capital to enhance the liquidity position of the bank. Alternatively, under a regime of mandatory reserve requirements, the CB would issue a certain number of "permits" required for a bank to accept a unit of deposit in real (resource) terms. See **Bernanke** and **Gertler** (1986) for more details.

behavior in this fashion is to capture some stylized facts about the structure of interest-free banks in the real world. The representative bank's resource constraint is given by:

$$L_b + E_b \;=\; (1 + d)\,K + R_b \tag{2}$$

Furthermore, let reserves of the banking system be proportional to PLS deposits and given by the following equation:

$$R_b \;=\; a\; L_b \tag{3}$$

where α ($0 < \alpha < 1$) is the reserve ratio against PLS deposits. Based on equation (3), equation (2) can be rearranged to solve for the level of investment K (where $K = 1 / (1 + \delta)\,[(1 - \alpha)\,L_b + E_b]$. Let the production function (in monetary terms) be $Y=Y(K, N, \theta)$, where N is units of labor, and è is a random variable representing the uncertain state of nature. For ease of computation, let the above production function (lower case denotes per capita terms) be:

$$y = \theta k^a \tag{4}$$

where a ($0 < a < 1$) is the marginal productivity of capital. Equation (4) is a neoclassical production function with constant return to scale, $y = Y/N$ is per capita output, and $k = K/N$ is per capita capital.[6] The above equation indicates that the only input is an endowment good which cannot be consumed immediately.

The riskiness of the PLS deposits induces the representative bank to plan its investment decisions in order to meet deposit and equity obligations. If the levels of investment and the amount of equity held by the CB are observable, the following individual rationality constraint (in per capita terms) must be binding:

$$(I - z)\,[E_t\, y(k, \theta) + r_b] \geq R_{PLS}\, l_b + R^e\, e_b \tag{5}$$

where z is the zakat rate, $E_t\; y\;(k,\; \theta)$ is the expected per capita gross return from investment, $r_b = R_b/N$ is per capita reserves held at the CB, $l_b = L_b/N$ is per capita deposits, and e, $= E_b/N$ is per capita equity. R_{PLS} and R^e are one plus the expected rates of return on PLS deposits and equity, respectively[7]. The expression to the right of inequality (5) is the cost of capital to the bank, while the expression to the left is the expected return net of zakat from both illiquid and liquid investments, respectively.

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6. For the random variable θ, $E(\theta)= 1$ whose standard deviation = $\sigma(\theta)$. Thus, $E(Y) = E(\theta)F(K)=F(K)$. Hence, for each realization of-`the state of the world , the production function is given by $\theta F(K)$; or, in per capita terms, by $\theta y(k)$.

7. The expected rates of return realized by PLS and equity holders, i.e., R_{PLS} and R^e, are calculated using the following formulae: $R_{PLS} = 1 + r^d$, where $r^d = (1\text{-}\ \lambda)II^*/L^*_b$ and $R^e = 1 + r^e$, where $r^e = (1\text{-}\ \lambda)II^*/E^*_b$, where II^* is the optimal net profit from all investments made by the bank. L^*_b, and E^*_b are respectively, optimal levels of PLS deposits and equity that maximize profits; and λ is the profit-sharing ratio accruing to the bank. Since the bank operates in a competitive environment, it considers R_{PLS} and R^e as costs of capital.

Inequality (5) implies that the bank would undertake an investment project only if it is potentially profitable (that is, expected returns should not be less than project costs). It further implies that realizing the expected return E_t, $y(k,\theta)$, and recovering required reserves r_b the bank pays zakat, deducts all costs, and then retains any remainder.

b. The Central Bank

The CB's assets comprise of equity in banks E_c; while its liabilities comprise of total reserves of banks, R_c, and total currency, M^S. Thus, the CB has the following balance sheet constraint (including capital account):

$$E_c = R_c + M^s \tag{6}$$

The right-hand side of equation (6) is the monetary base, while the left-hand side is its assets (equity of banks).

c. The Household

As stated above, households allocate their initial wealth between PLS deposits and real money balances, while abiding by their budget constraint. The excess of the proceeds from investment over consumption is saved. The representative household maximizes its time-invariant utility $U(c_t, m_t)$, which includes as arguments, in per capita terms, real consumption (c) and real money balances (m). Real balances enter positively the utility function since money reduces transaction costs [see Fischer (1979) and Gertler and Granules (1981)]. We assume that the utility function is strictly concave with twice continuously differentiable, and that both commodities are non-inferior [see Sidrauski (1967)]. The household's end of period income consists of returns from holding deposits at financial institutions. Since there is no fixed rate of return on assets, wealth accumulates through returns from PLS deposits at banks. Hoarding cash in banks entails a negative nominal rate of return equal to the zakat rate. The representative household's future income (in per capita terms) can be formulated as follows:

$$\dot{D}_t \,/\, N + \dot{M}_t \,/\, NP_t \quad = \quad (1+R_{PLS})\,(1-z)\,dt + (1-z)\,m_t$$

where $\dot{D}_t / \mathrm{N}$ and $\dot{M}_t / \mathrm{NP}_t$ are respectively the per capita growth rates of deposits and real cash balances, (N is population, P is the price level, and the dots over the variables denote their respective rates of growth). At any moment, disposable income should equal consumption plus saving [i.e., $(1+R_{PLS})\,(1-z)\,d_t + (1-z)\,m_t = c_t + s_t$]. Expressing per capita asset holdings as $w_t = d_t + m_t$, we can write the budget constraint in real terms as (see Appendix A for details):

$$\dot{w} = (1-z)\,(1+R_{PLS})\,w_t - n\,w_t - R_t\,m_t - c_t \tag{8}$$

where $R = [\pi + (1 - z) R_{PLS}]$ is the nominal rate of return in the economy, n is the rate of growth of population $(n = \dot{N})$ and π is the inflation rate.

Under a general equilibrium framework, all markets clear and a balance-sheet equilibrium equation can be written as:

$$(D_t + M^d - W) = [(1 + \delta) K_t + R_b - L_b - E_b] + (R_e - M^3 - E_c) \tag{9}$$

This adding-up economic requirement has an interesting implication in each market equilibrium dictates that the demand for assets equals the supply. However, the three markets (i.e. capital, equity and money) are not independent of each other since aggregate assets sum up to aggregate liabilities. To determine the equilibrium rates of return, we need to drop one market as redundant. Since the nominal (own) rate of return on money is assumed zero, and zakat is institutionally fixed we may choose to drop the money market[8]. Equilibrium in the various asset markers may be represented by the following system of equations:

Money:

$$M^d = M^S \tag{10.1}$$

Reserves:

$$R^c = R_b \tag{10.2}$$

Equity:

$$E_c = E_b \tag{10.3}$$

Capital (PLS):

$$D_h = L_b \tag{10.4}$$

and the total initial endowments equal investment, that is:

$$W_t = (1 + \delta)K_t \tag{10.5}$$

Equation (10.4) shows that the amount of PLS deposits that banks wish to accept at any PLS rate should be equal to the quantity of deposits that the public desire to hold at this same rate. On the other hand equation (10.5) indicates that the total amount of investment must equal total endowments in the economy. Given that the rate of return on money is zero, there are only two endogenously determined variables; namely, the rate of return on PLS deposits (R_{PLS}), and the rate of return on equity (R^e).

8. If the CB has effective control over its liabilities (i.e. currencv and banks' reserves), we can use the familiar Walras' Law to eliminate either of these two markets. Note further that effectiveness can be measured by the magnitude of the response of some crucial variables, for example, the rate of return on PLS deposits or money holdings, to a change in a policy variable like the reserve ratio.

Since the bank's demand for the two assets (PLS deposits and equity) hinges on the above two rates, the system equilibrium is reached through adjusting these rates. The fact that such rates are endogenous and flexible should enable the CB to use its instruments of monetary control to influence aggregate output and induce the market rates of return towards their desired levels. Using the above system of equations to solve for R_{PLD} and R^e, we first need to formulate the production possibilities of the banks, and the preferences of households. We address this task next.

IV. Model Solution and Implications

a. The Bank's Maximization Problem

Given the reserve requirement ratio α, each bank's objective is to choose the sequence of capital and equity (k_t, E_b) that would maximize the intertemporal sum of expected profits:

$$E_t\left(\sum_{t=i}^{n} II_t\right) \tag{11}$$

subject to (2), (3) and (4), where E_t is the expectations operator conditional on the information set at time t, and

$$II_t = (1-z)\,Ey\,(k,\theta) + (1-z)\,r_b - R_{PLS}\,l_b - R^e\,e_b \tag{12}$$

The first terms on the right side of expression (12) reflects gross returns from the bank's investment. The second term is the amount held in pursuance of reserve requirement net of zakat. The third and the fourth terms represent the bank's obligations. Equation (l2) states that bank's profit comes in the form of the output good which cannot be reinvested. This assumption greatly simplifies the analysis by making the bank's sequential decisions independent of the returns to earlier investments. The assumption also makes it unnecessary to introduce an arbitrary discount factor in (12) to bound the accumulation of wealth.

To solve the bank's profit-maximization problem stated in (12), we use the resource constraint (2), the reserve equation (3), and the equation for K to eliminate bank deposits (l_b) and reserve (r_b) from the problem. The first order necessary conditions for optimality are:

$$\partial II/\partial k = (1-z)\,\partial E_t\,y\,(k,\theta)/\delta k + ((1+\delta)/(1-\alpha))[(1-z)\,\alpha - R_{PLS}] = 0 \tag{13}$$

$$\partial \text{II} / \partial e_b = (1-z)\,\partial E_t\, y(k, \theta) / \delta k \; dk / de_b \; + ((1+\delta)/(1-\alpha))[(1-z)$$
$$\alpha - R_{PLS}]\, dk / de_b \; - (1/(1-\alpha))\,[(1-z)\,\alpha - R_{PLS}] - R^e = 0 \tag{14}$$

where ∂E_t k (k, θ)/∂k is the expected marginal productivity of capital, and dk/de_b is the marginal effect of increasing equity on capital stock.

Given the values of y, δ, α, and z, then equations (13) and (l4), together with equilibrium equations (10.3) and (10.4), can be solved for the equilibrium rates of return:

$$R^*_{PLS} = ((1-z)\,(1-\alpha)/(1+\delta)) / \partial\, E_t\, y(k, \theta) / \partial k + (1-z)\,\alpha \tag{15}$$

$$R^{*e} = ((1-z)/(1-\delta)\,\partial\, E_t\, y(k, \theta) / \partial k \tag{16}$$

Given the restrictions imposed on the marginal productivity of capital, a [see equation 4], the reserve ratio, a. the zakat rate, z, and the intermediation cost, δ; both equilibrium rates are positively signed. Equations (15) and (16) show that both rates depend on the marginal productivity of capital. Furthermore, equation (15) also implies that changes m the reserve ratio positively affect the equilibrium rate of return on PLS deposits.

b. The Household Choice Problem

Given the optimal rate of return on PLS deposits, the representative household (with perfect foresight) solves the following lifetime utility function:

$$U \;=\; \int_0^e e^{-(\rho-n)t}\, u(c_t, m_t)\, dt \tag{17}$$

subject to (8) above, where ρ (ρ > 0) is the fixed subjective rate of time preference, and *n* is the rate of growth of population. The above utility function is assumed to be strictly concave and twice continuously differentiable. In order to achieve a closed form solution, we assume the utility function to be of the following form (see Fischer, 1979):

$$u(c_t, m_t) = [c_t^v\, m^{\beta(\lambda)}]^{1-\sigma} / (1-\sigma) \tag{18}$$

where σ > 0; and v > 0, β > 0 are the marginal utilities of consumption and money, respectively, and β'(λ) < 0[9].

To solve the above maximization problem, we can set the Hamiltonian:

9. To postulate a decreasing marginal utility of holding monev with respect to λ is reasonable since high λ is expected to induce economic agents to hold less real cash balances. Note also that when σ=l, the logarithmic utility function becomes = v ln c_t + β lm m_t.

$$H = e^{-(\rho-n)t}\{[c_t^v\, m^{\beta(\lambda)}]^{1-\sigma}/(1-\sigma) + r_t[(1-z)(1+R_{PLS})w_t - n\,w_t - R_t m_t - c_t]\} \quad (19)$$

where r_t is the Lagrangian multiplier for the household budget constraint. It can be seen that this problem leads to the following solutions:

$$e^{-(\rho-n)t}[c_t^v\, m^{\beta(\lambda)}]^{-\sigma}\ c_t^{v-1}\, m^{\beta} - r_t \quad = \quad 0 \qquad (20)$$

$$e^{-(\rho-n)t}[c_t^v\, m^{\beta(\lambda)}]^{-\sigma}\ c_t^{v-1}\, m^{\beta} - r_t\ R_t \ = \quad 0 \qquad (21)$$

$$\lim_{t-0}\ r_t\ w_t\ e^{-(\rho-n)t} \quad = \quad 0 \qquad (22)$$

Equations (20) and (21) imply the following familiar result:

$$\beta\, c_t\, /\, v\, m_t \ = \ \ R_t \qquad (23)$$

At the optimum, the marginal utility of money standardized in terms of the marginal utility of consumption equals the nominal rate of return in the economy. Alternatively, optimal risk sharing would enable the consumer to equate the ratio of marginal utilities to the marginal rate of return. This equation further suggests that the demand for real balances is inversely related to the nominal rate of return. This equation further suggests a sequential pattern to the household's choice problem. Given the nominal rate of return, the representative household chooses the optimal levels of consumption and real balances to maximize its lifetime utility function.

Equations (20), (21), and (23) can then be solved to obtain the following demand functions (see Sidrauski, 1967):

$$c_t^d = vR_t\, m_t\, /\, \beta = c^d\ (\pi, y, w, \alpha; z) \qquad (24)$$

$$m_t^d = \beta\, c_t\, /\, vR_t\ = m^d\ (\pi, y, w, \alpha; z) \qquad (25)$$

Given the zakat rate (z), it can be seen that the households' demand for consumption and real balances are affected by the following factors: the rate of inflation (π), real output (y), real wealth (w), and the reserve ratio (α). Thus, given the optimal choice of real balances and consumption, the representative household allocates the remainder of their wealth to PLS deposits.

c. Changes in the Reserve Ratio

Equations (15) and (16) above indicate that changes in the reserve ratio directly affect the equilibrium rate of return on PLS deposits. However, the reserve ratio only indirectly affects the rate of return on equity. Recall that an increase in the reserve ratio always reduces the amount of funds available for investment and decreases (or at least leaves unaffected) the level of output[10].

To examine the way in which changes in the reserve ratios can alter the effectiveness of monetary control, we need to impose the following restriction either on the marginal productivity of capital or on the expected rate of return from PLS deposits (see Appendix B):

$$(1/(1+\delta))\,\partial\, E_t\; y(k,\theta)/\partial k > 1 \tag{26}$$

The expression to the left of inequality (2) represents the expected return from PLS deposits. This inequality is an individual rationality constraint which implies that households prefer holding PLS deposits exclusively to holding liquid assets as long as the expected rate of return from PLS deposits exceeds that from liquid assets.

`To address the response of R_{PLS} and R^e to changes in the reserve ratio, we differentiate (15) and (16) with respect to α. We get:

$$\begin{aligned}\partial R_{PLS}/\partial\alpha = ((1-z)/(1+\delta))[(1+\delta) - \partial\, E_t\, y(k,\theta)/\partial k \\ + (1+\alpha)\,\partial^2\; E_t\, y(k,\theta)/\partial k^2\; dk/d\alpha]\end{aligned} \tag{27}$$

$$\partial R^e/\partial\alpha = ((1-z)\,(1+\alpha))\,\partial^2\; y(k,\theta)/\partial k^2\; dk/d\alpha] \tag{28}$$

Observe that the third term in the bracket in equation (27) is positive. If the return constraint (26) is binding, then the sum of the first and second terms in (27) is negative. Equation (27) then states that an increase in the reserve ratio has two opposite effects on the rate of return on PLS deposits. That is, on the one hand, it increases the demand for liquid assets [see equation 29 below]. But since the rate of return on liquid assets is less than the expected return on investment [that is, ([1-(1/1+δ)∂ $R_{PLS}/\partial\alpha$] < 0), it thus reduces R_{PLS}. On the other hand, it reduces the amount of capital available for investment ($dk/d\alpha < 0$) and therefore reduces the marginal productivity of capital ($\partial^2\, E_t y/\partial k^2 < 0$), thereby increasing R_{PLS}.

Stability in the model necessitates that the rate of return on PLS deposits be a decreasing function of the reserve ratio, that is, $\partial\, R_{PLS}/\partial\alpha < 0$. Furthermore, given that $dk/d\alpha < 0$, equation (28) reveals that increasing α would have a positive effect on the rate of return on equity, that is, $\partial R^e/\partial\alpha < 0$. Note also that an increase in the reserve ratio should reduce the amount of capital available for investment. But since the bank's

10. **Brainard** (1967) contains a useful account of the effects of changes in the reserve ratio on assets' rates of return.

capital consists of PLS deposits and equity, the CS can offset the shortage in liquidity by injecting new reserves in order to increase its equity stake. This would increase the level of output, and eventually increase the rate of return on equity. Hence the CB can effectively influence the level of economic activity by influencing the amount of liquidity available for investment.

To discuss the effect of changes in the reserve ratio on the demand for money, differentiate equation (25) with respect to α. We get:

$$\partial m / \partial \alpha = -\beta c_t / vR^2 \, \partial R / \partial \alpha \tag{29}$$

Both terms on the right-hand side of (29) are negative, implying that the demand for money is a positive function in the reserve ratio, that is, $\partial m/\partial\alpha > 0$ (see Appendix C). Equations (27) and (29) suggest that a higher reserve ratio reflects expansionary monetary policy since it lowers the rate of return on PLS deposits and induces the public to hold more cash balances[11].

d. Changes in the Profit-Sharing Ratio

Now suppose that the production function is linear in the capital stock such that the per capita production function can be written as: $E_t\, y(k, \alpha) = \phi(\lambda)k$, where $\lambda(0 < \lambda < 1)$ is the profit-sharing ratio accruing to the bank. The quantity $\phi(\lambda)$ can be interpreted as a learning-by-doing technology improvement resulting from research and development. It is reasonable to assume that the accumulated knowledge stimulates the marginal productivity of capital. We further assume that increasing the profit-sharing ratio would also improve the state of knowledge. That is, $\phi'(\lambda) > 0$[12]. By influencing the profit-sharing ratio, the CB can impact not only innovations in the banking system, but also the rates of return in the economy[13]. This can be seen by substituting for the value of $E_t\, y(k, \theta)$ in both (15) and (16) and differentiating with respect to λ:

$$\partial R_{PLS} / \partial \lambda = ((1-\alpha)(1-z)/(1+\delta))\phi'(\lambda) \tag{30}$$

$$\partial R^e / \partial \lambda = ((1-\alpha)/(1+\delta))\phi'(\lambda) \tag{31}$$

11. Following **Tobin** and **Brainard** (1967), a given monetary policy is considered expansionary (restrictive) if it lowers (raises) the rate of return on financial assets (PLS deposits). This is analogous to the familiar IS/LM apparatus in the context of an interest-based economy.

12. The learning-by-doing process is injected here to underscore the need for Islamic banks to invest in research and development in order to improve their banking techniques and sharpen their methods of mobilizing funds.

13. Although the profit-sharing ratio is determined through negotiations between the bank and its partners, the CB can set the upper and the lower bounds for the ratio, depending on the state of the economy.

Observe that both expressions (30) and /31) are positive. This implies that the CB can effectively use the profit-sharing ratio as a policy tool, at least within bounds, to change the amount of funds available for investment in the economy. The profit-sharing ratio can also be used to regulate the demand for real money. To see that, differentiate equation (25) with respect to λ to get:

$$\partial m_t / \partial\lambda = \beta'(\lambda)\, c_t / vR_t \tag{32}$$

Given $\beta'(\lambda) < 0$, then $\partial m_t/\partial\lambda$ is negatively signed; that is, a higher profit-sharing ratio reduces money demand. The interpretation of this theoretical result is straightforward since higher λ raises the rate of return on PLS deposits and thus encourage the public to move out of cash and into PLS deposits. Given that a high profit-sharing ratio increases the rates of return on PLS deposits but reduces the demand for cash balances. it can be seen that a high profit-sharing ratio has contractionary effects on the economy.

V. Concluding Remarks

The viability of an economic system based on equity participation instead of a fixed rate of interest requires the design of reliable and effective monetary control tools to replace those of the interest-based system. The Central Bank in an Islamic economy plays a pivotal role in the transformation process. Among other things, the Central Bank should utilize innovative techniques to control monetary aggregates. Although most of the relevant issues are hard to model analytically, the model developed here is suggestive and shows that the Central Bank in an Islamic economy can effectively exercise monetary control by manipulating three basic instruments; namely, reserve ratio, the profit-sharing ratio, and equity holdings of banks. By finessing these instruments, the Central Bank can achieve its intermediate target of pegging the equilibrium rates of return in the economy. Contrary to standard interest-based models, our results from the interest-free model imply that the Central Bank can stimulate real money demand by raising the reserve ratio for banks. This is because increasing the reserve ratio reduces the rate of return on PLS deposits. thus discouraging the public from supplying funds to banks, and eventually reducing the amount of funds available to finance investment. Given the zakat rate , the public would react by holding more cash balances. On the other hand a high profit-sharing ratio is contractionary since it increases the rates of returns on PLS deposits and reduces the demand for real balances. While the Central Bank cannot directly control the profit-sharing ratio, it can nevertheless peg it effectively.

Of course, the above conclusions are only suggestive and predicated on the model assumptions. Several important issues are also neglected in this paper and may prove useful for future research. Among them are the ways in which the government can monetize its deficit and how expectations about the rate of returns are formulated. In addition, our modeling efforts do not integrate uncertainty into the analysis or elaborate on inflation and its efforts on the real sector.

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Appendix A

To derive equation (8) in the text, note that the budget equation can be written as

$$(1 - z)\,(1 + R_{PLS})\,d_t + (1 - z)\,m_t = c_t, + s_t \tag{A.1}$$

indicating that, at all times, disposable income must equal real consumption (c_t) plus press real saving (s_t). Following Sidrauski (1967), gross real saving is equal to gross capital formation (i_t) plus the gross addition to money holding (x_t); that is:

$$s_t = i_t + x_t \tag{A.2}$$

where

$$i_t = \dot{d}_t + nd_t \tag{A.3}$$

$$x_t = \dot{m}_t + (n + p)m_t \tag{A.4}$$

Equation (A.3) indicates that gross capital formation is equal to the growth in PLS deposits ($\dot{d}_t$) plus the amount of capital needed to provide for the newly born members of the household ($n\,d_t$). Equation (A.4), on the other hand, indicates that the press addition to money holding can be decomposed into the rate of growth of real balances ($\dot{m}_t$) plus the amount needed for the new born ($n\pi_t$), plus the amount needed to offset inflation (πn_t). Substituting (A.2), (A.3), and (A.4) in (A. 1), we have:

$$(1-z)(1+R_{PLS})d_t + (1-z)\,m_t - \dot{d}_t - nd_t - \dot{m}_t - (n+\pi)\,m_t - c_t, \tag{A.5}$$

Equation (A.5) can be written as:

$$\begin{aligned}(\dot{d}_t + \dot{m}_t) = {} & (1-z)(1+R_{PLS})(d_t + m_t) - (1-z)(1+R_{PLS})m_t \\ & + (1-z)m_t - n(d_t + m_t) - \pi m_t - c_t\end{aligned} \tag{A.6}$$

Using the notation $w_t = d_t + m_t$, and $R_t = \pi + (l\text{-}z)R_{PLS}$, equation (A.6) can be rewritten as:

$$\dot{w}_t = (1-z)(1+R_{PLS})\,w_t - nw_t - R_t\,m_t - c_t \tag{A.7}$$

Appendix B

To derive restriction (26) in the text, recall that the household has a choice between investing in PLS deposits and reap the expected return (R_{PLS}), or keeping their wealth in real balances and get the return (l-z). Individual rationality implies:

$$R_{PLS} > (1\text{-}z) \tag{B.1}$$

substituting the value of the expected rate of return, we have:

$$R_{PLS} = ((1-z)\,(1-\alpha)/(1+\delta))\partial E_t\, y(k, \theta)/\partial k + (1-z)\,\alpha > (1-z) \quad (B.2)$$

which indicates:

$$(1-z)\,[(1-\alpha)/(1+\delta))\partial E_t\, y(k, \theta)/\partial k + \alpha > (1-z)] \quad (B.3)$$

Dividing both sides of the inequality by (1-z), equation (B.3) can be rewritten as:

$$(1-\alpha)/(1+\delta)\partial E_t\, y(k, \theta)/\partial k + \alpha > 1 \quad (B.4)$$

Now, subtracting α from both sides of the inequality, we get:

$$(1-\alpha)/(1+\delta)\partial E_t\, y(k, \theta)/\partial k > (1-\alpha) \quad (B.5)$$

Equation (B.5) can further be reduced to:

$$(1/(1+\delta))\partial E_t\, y(k, \theta)/\partial k\ 1 \quad (B.5)$$

Appendix C

To prove the positive sign for equation (29), note that $R_t = \pi + (1-z)\, R_{PLS}$, where

$$R_{PLS} = (1/1+\delta)\,[(1-\alpha)\, l_b + e_b].$$

Now, taking the derivative of the money demand equation (25) with respect to α we get:

$$\partial m / \partial\alpha = \partial m_t / \partial R \,.\, \partial R / \partial\alpha \quad (C.1)$$

where

$$\partial R / \partial\alpha = -\,((1-z) / (1 + \delta))\, l_b \quad (C.2)$$

It can be easily seen that $\partial R / \partial\alpha < 0$. But since $\partial m_t / \partial R < 0$, then the right hand side of (C.1) is positive.

إدارة النقود في اقتصاد خال من الفائدة

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المستخلص : يناقش لبحث دور السياسة النقدية في اقتصاد خالٍ من الفائدة عن طريق تطوير نموذج توازني عام. إن الأدوات التي في مقدور البنك المركزي التحكم من خلالها في عرض النقود في الاقتصاد تتركز في : نسبة الاحتياطي، نسب تقاسم الربح بين الشركاء ونسبة الملكية ضمن خصوم البنك. بهذا يستطيع البنك المركزي أن يتحكم ضمن نطاق معين في معدلات الفائدة التوازنية في الاقتصاد. كذلك يستطيع البنك المركزي أن يحفز الطلب الحقيقي على النقود من خلال زيادة نسبة الاحتياطي المطلوبة من البنوك. أما في حالة الرغبة في تخفيض الطلب على النقود فإن نسبة تقاسم الربح بين الشركاء يمكن زيادتها حتى ولو كان ذلك بطريقة غير مباشرة. إن النموذج المقترح لا يقوم بإدراج حالة عدم التأكد في التحليل أو مناقشة تأثير التضخم على القطاع الحقيقي في الاقتصاد بشكل مفصل.